Higher spin mapping class groups and applications

Nick Salter
Joint with Aaron Calderon
Columbia University
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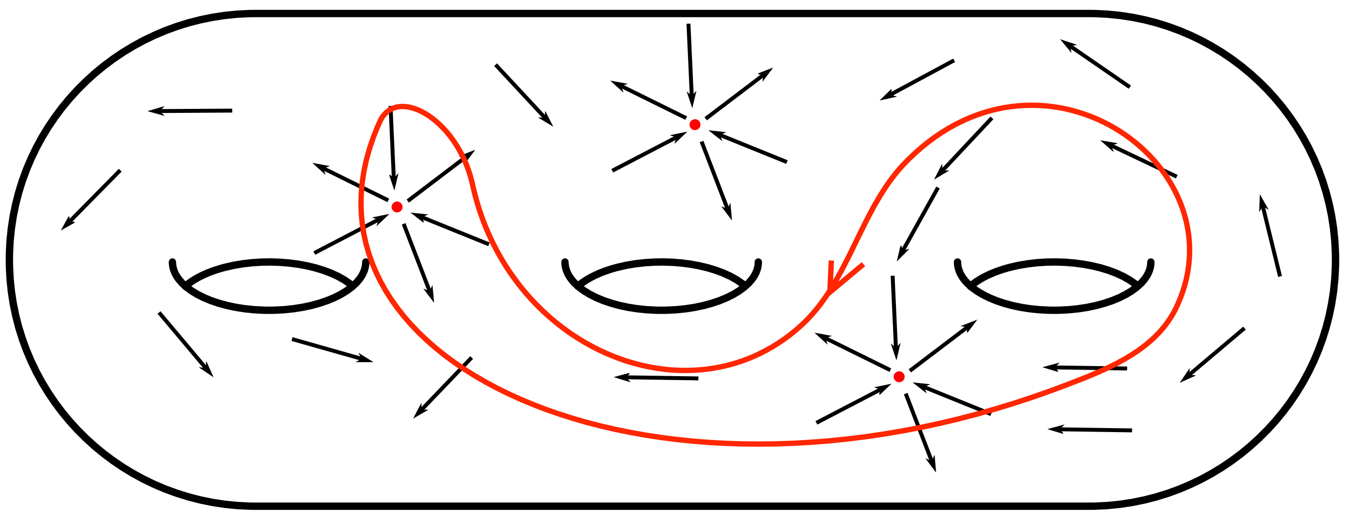
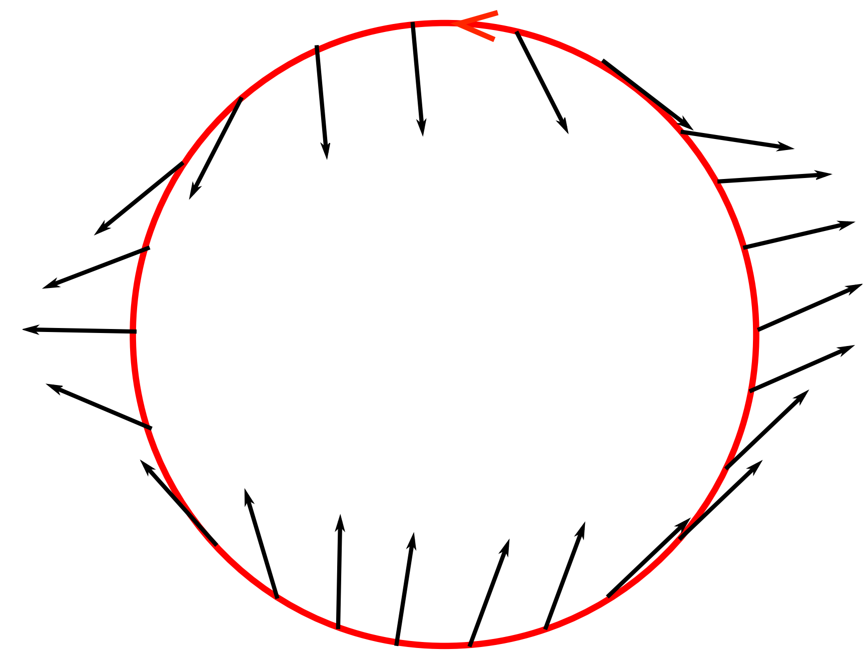
Higher spin structures

Paradigmatic example:





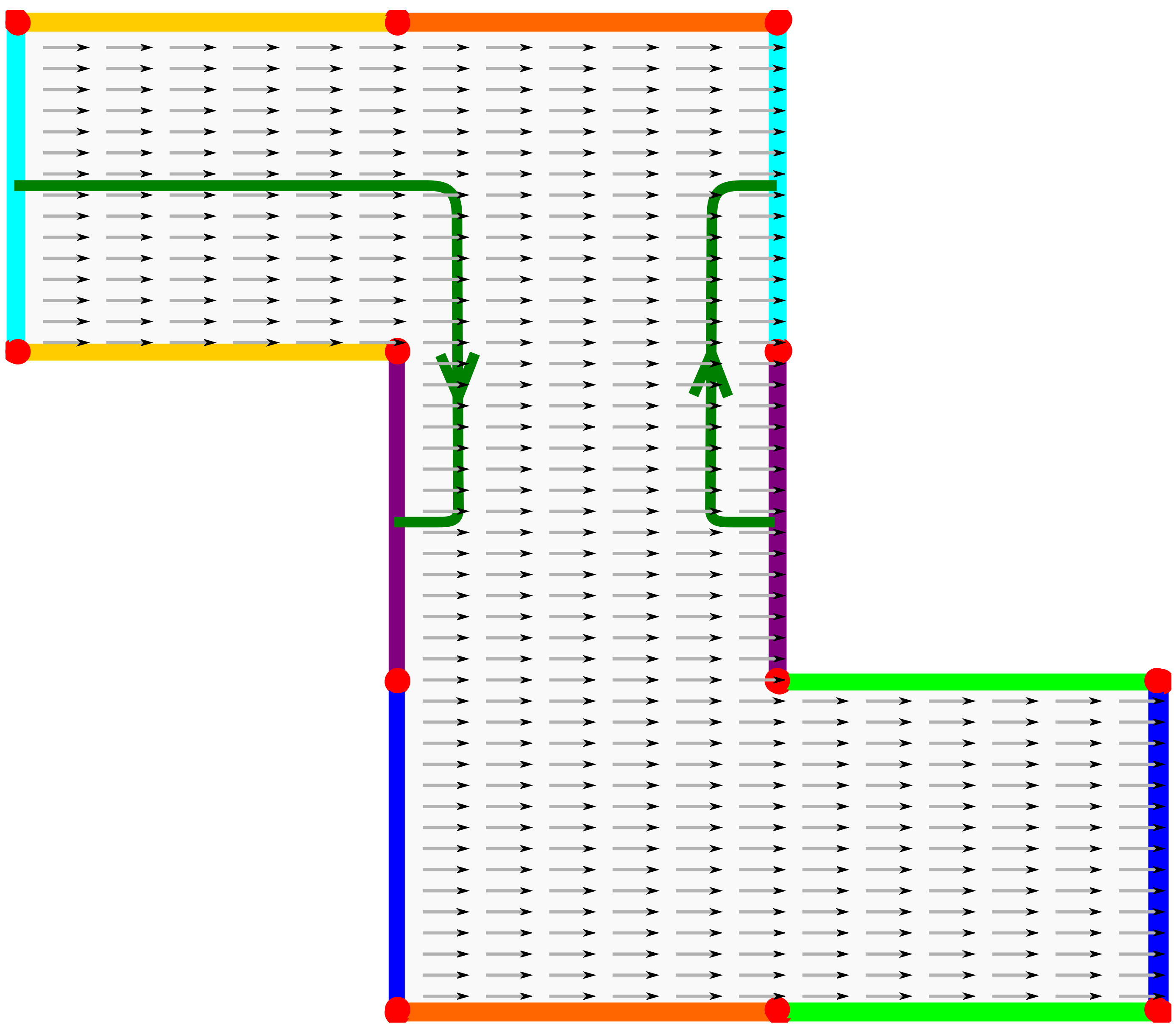
$$\phi: \begin{Bmatrix} \text{oriented SCC's} \\ \text{on } \Sigma \end{Bmatrix}_{/\text{isotopy}} \longrightarrow \mathbb{Z}/r\mathbb{Z}$$

$$c \longrightarrow \text{Winding number of c rel } \xi$$

Such ϕ: r-*spin structures*.

Also called r-*winding-number functions*.

Translation surfaces



Strata

$\kappa = \{\kappa_1, \ldots, \kappa_n\}$: partition of 2g-2

Stratum $\mathscr{H}\Omega(\kappa)$: all translation surfaces with cone angle set κ

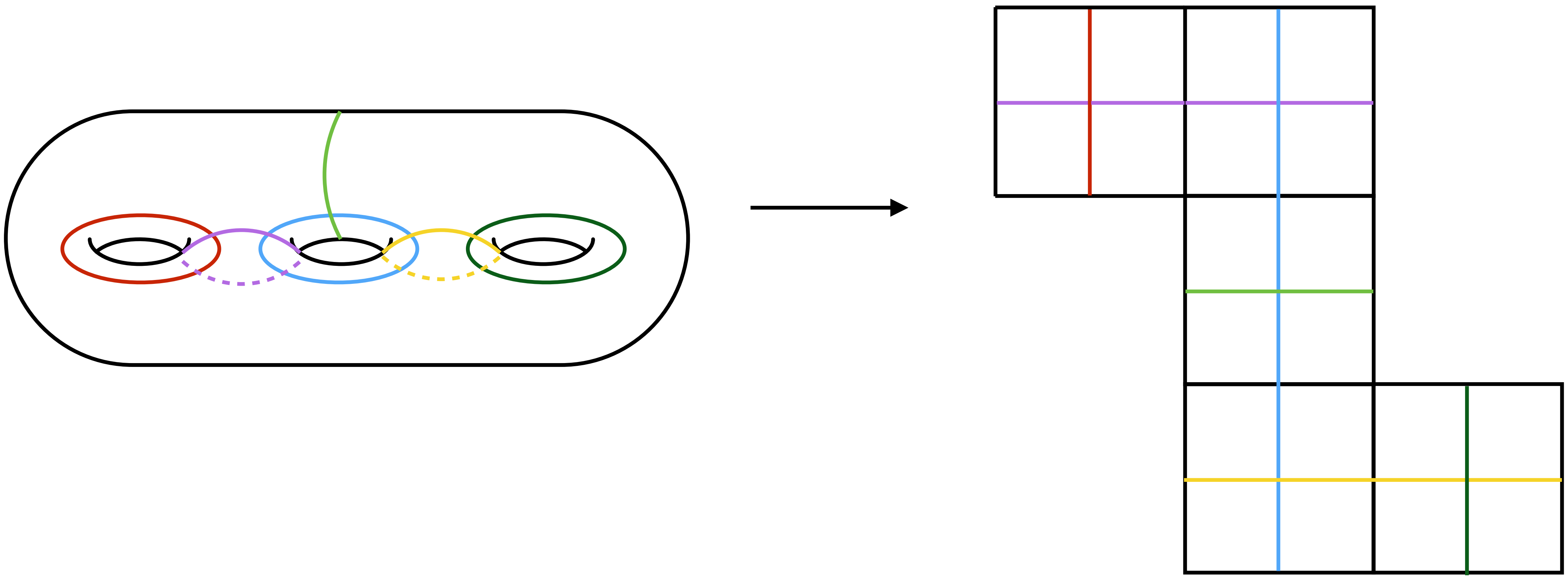
Basic problem: understand the topology of $\mathscr{H}\Omega(\kappa)$

$\pi_0(\mathscr{H}\Omega(\kappa))$ understood (Kontsevich-Zorich). Always $\leq$ 3 components.

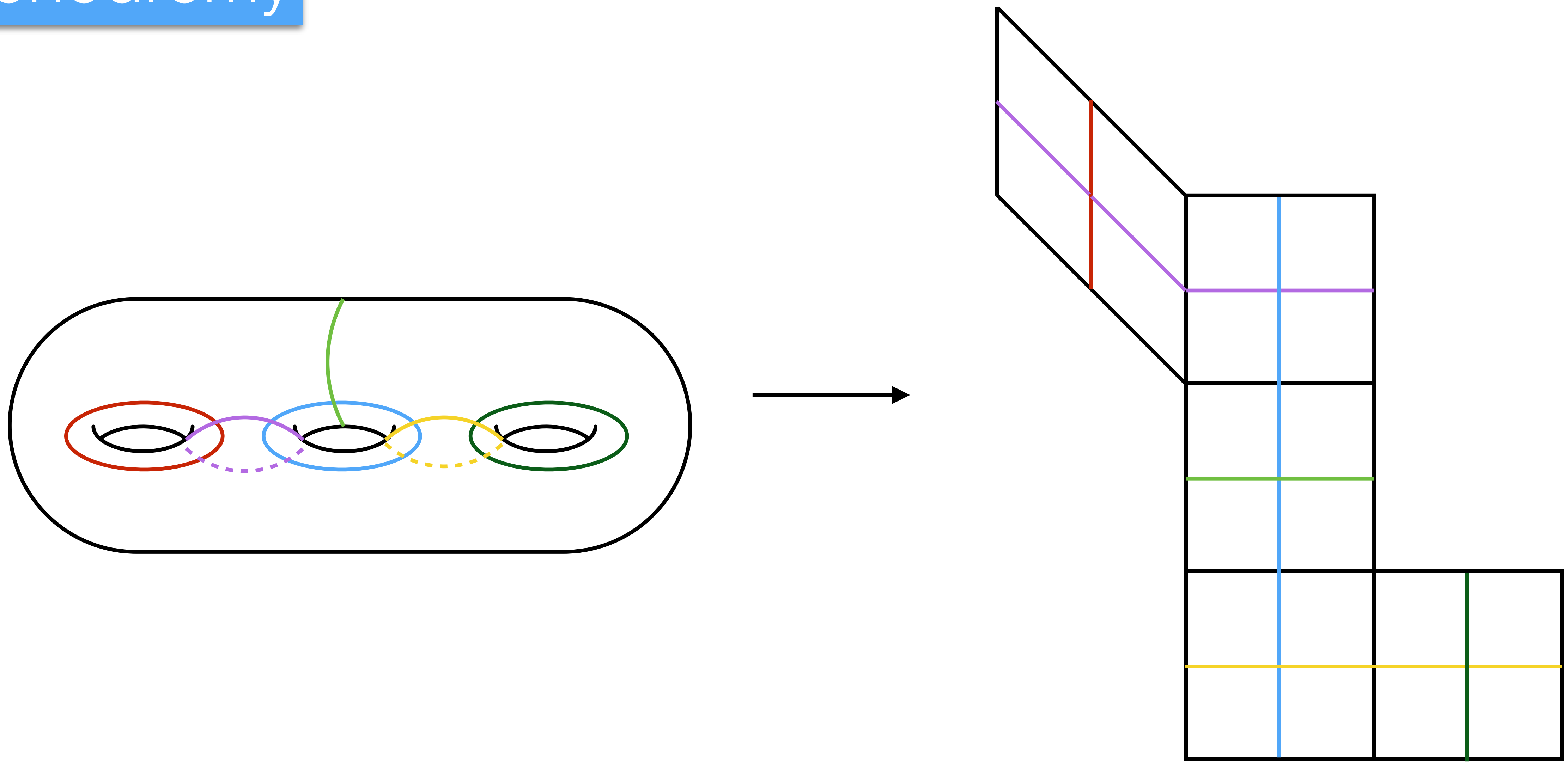
Fix $\mathscr{H} \subseteq \mathscr{H}\Omega(\kappa)$. What is $\pi_1(\mathscr{H})$?

Turns out r-spin structures play an important role in this question!

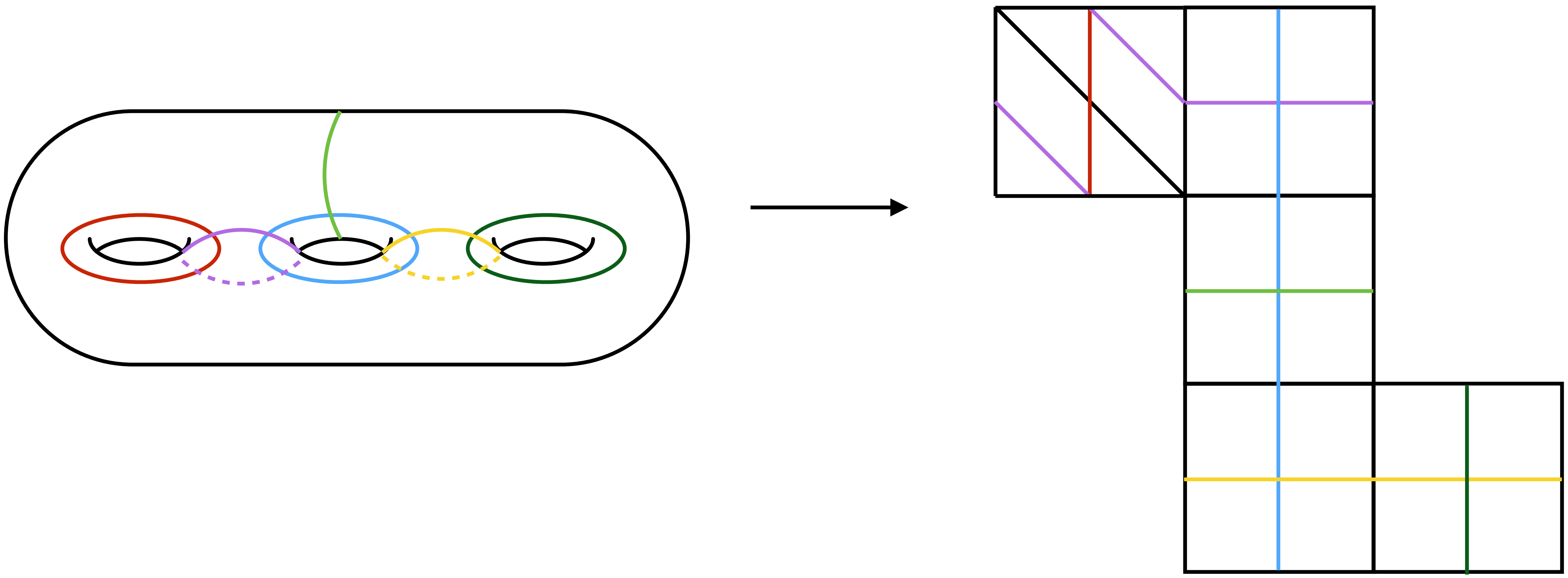
Monodromy



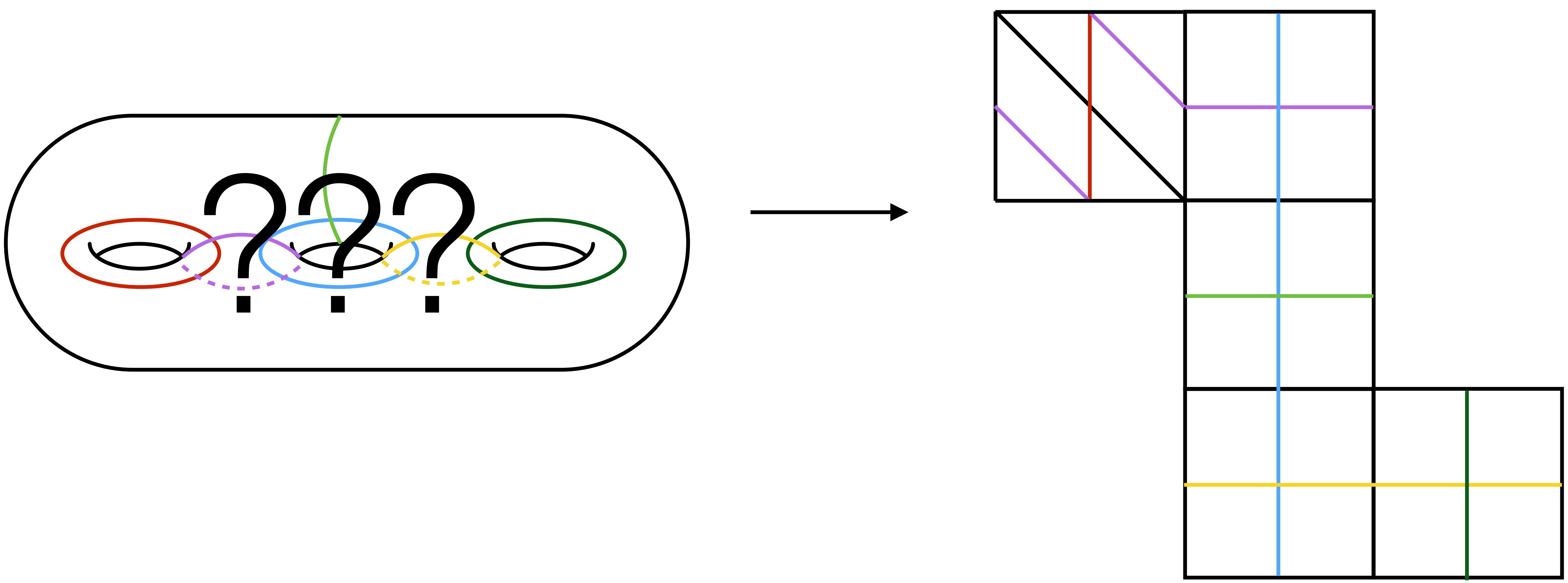
Monodromy



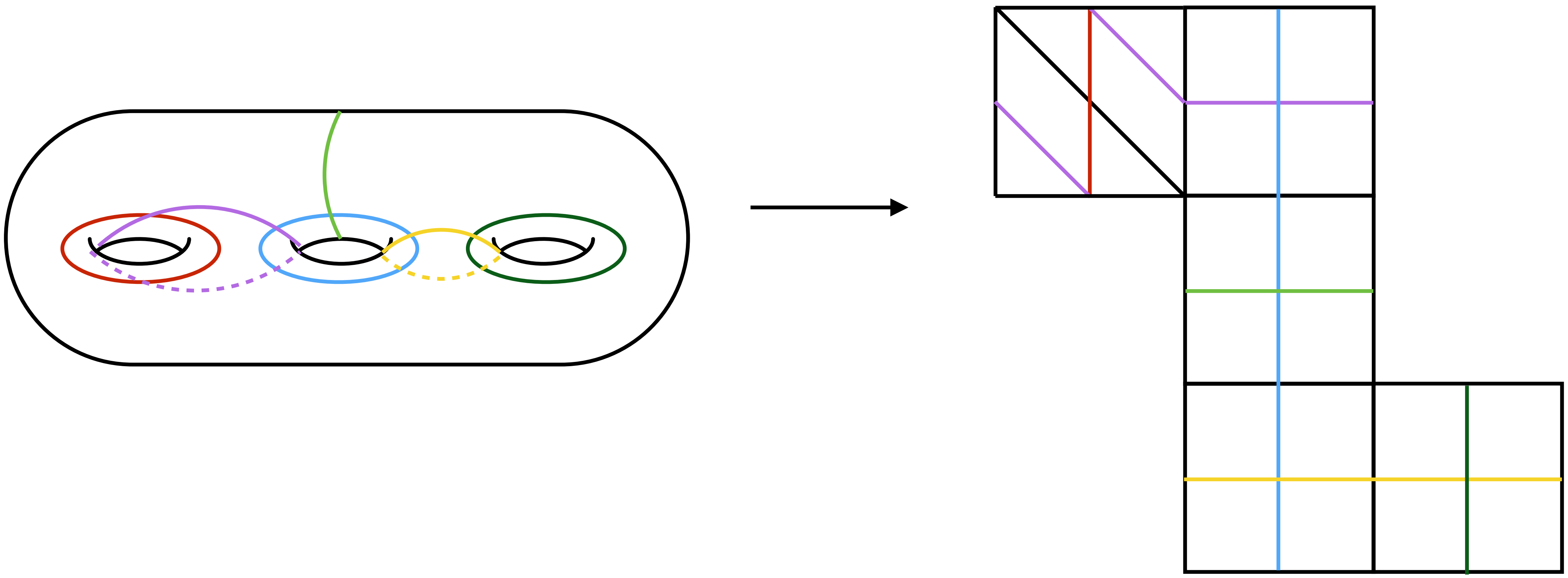
Monodromy



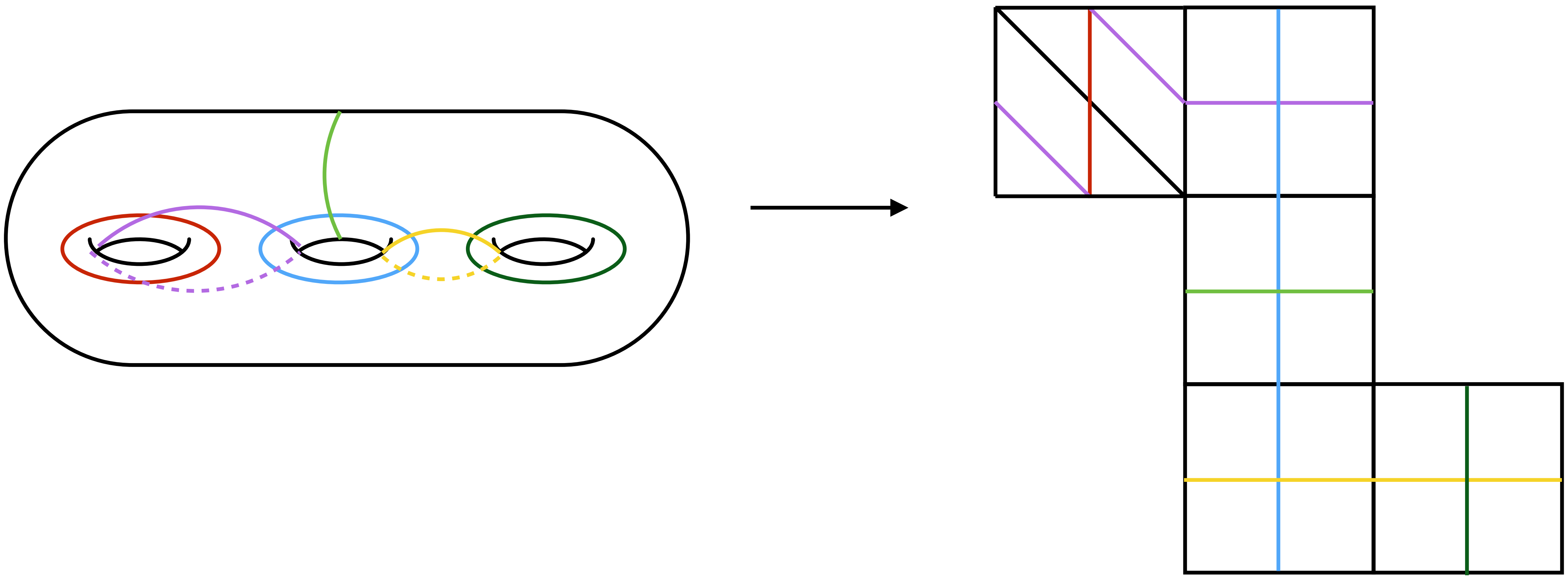
Monodromy



Monodromy



Monodromy



Gives us map $\rho_{\mathscr{H}} : \pi_1(\mathscr{H}) \to \mathbf{Mod}(\Sigma_g)$

- A method to study this mysterious group
- Tells us about translation surfaces

What does the presence of a spin structure tell us?

Higher spin mapping class groups

$\mathrm{Mod}(\Sigma_g)$ acts on set of r-spin structures (precomposition)

$\mathrm{Mod}(\Sigma_g)[\phi]$: stabilizer of ϕ

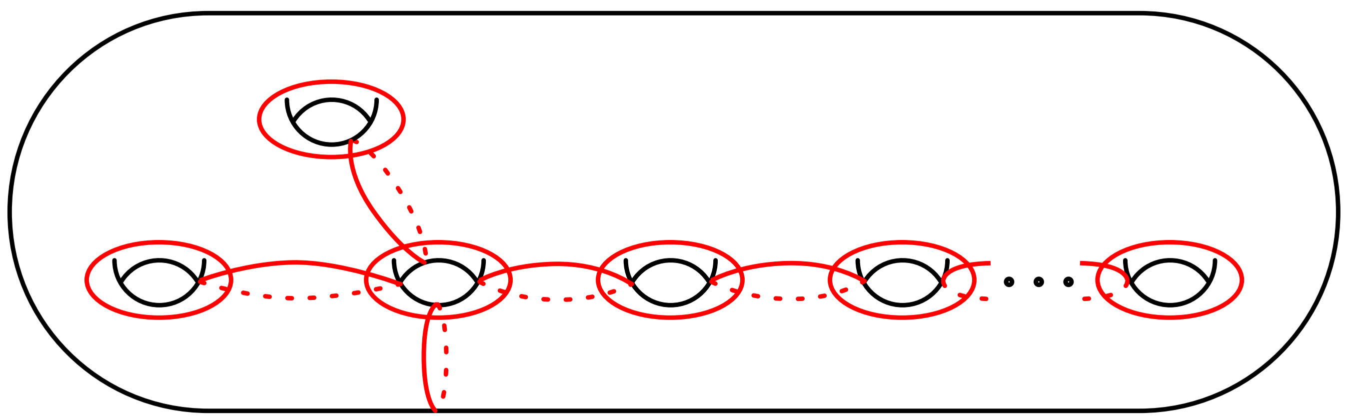
Invariant horizontal vector field —> invariant r-spin structure ϕ

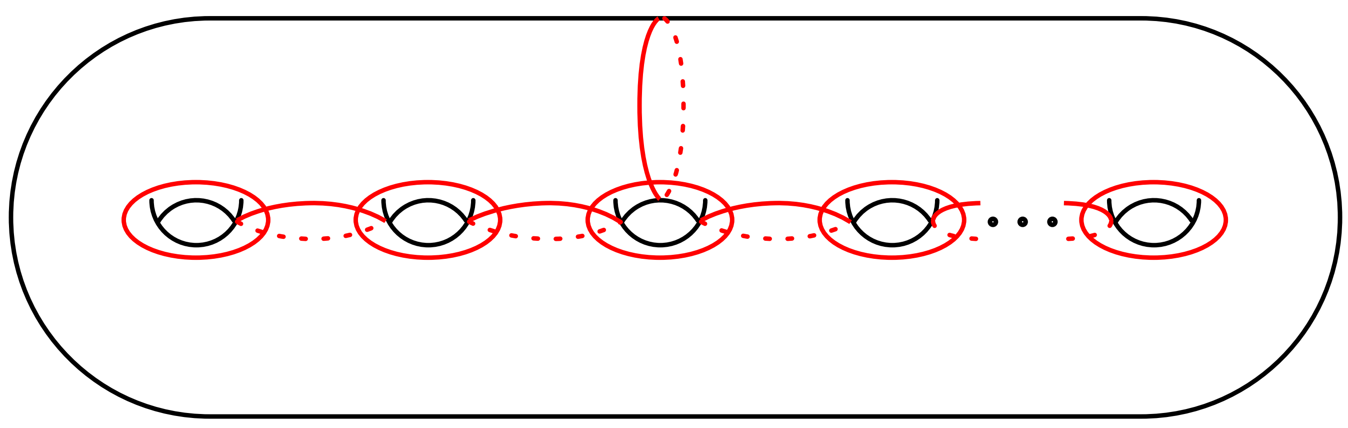
$$\rho_{\mathscr{H}} : \pi_1(\mathscr{H}) \to \mathrm{Mod}(\Sigma_g)[\phi]$$

To study $\rho_{\mathscr{H}}$, must understand $\mathrm{Mod}(\Sigma_g)[\phi]$!

Simple generating sets

Theorem (Calderon - S.): For g ≥ 5, any r-spin structure ϕ, there is an *explicit* finite generating set for $\mathrm{Mod}(\Sigma_g)[\phi]$ consisting of Dehn twists.





Applications

Theorem (Calderon - S.): Fix g ≥ 5, κ partition of 2g-2, and $\mathscr{H} \subseteq \mathscr{H}\Omega(\kappa)$ "non-hyperelliptic"*. Then

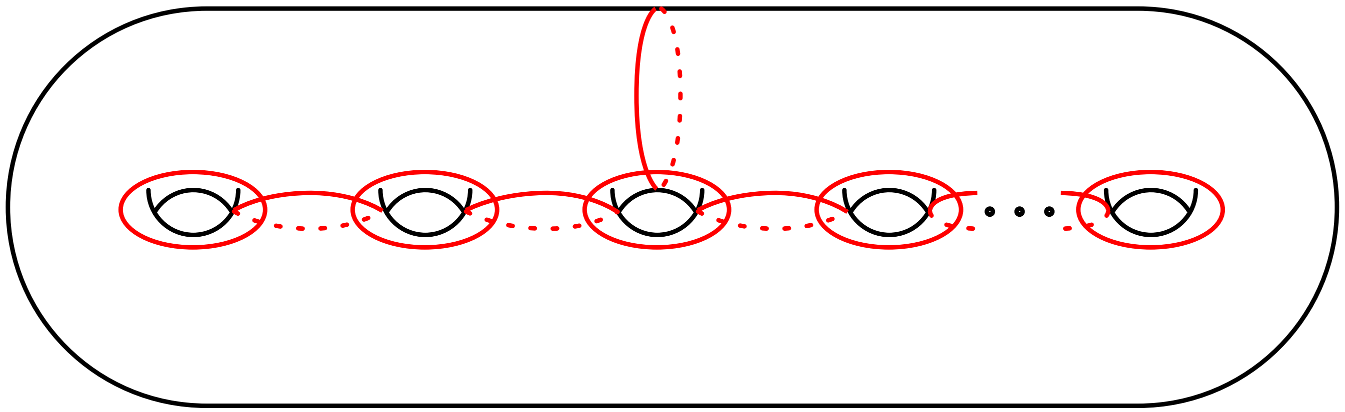
$$\rho_{\mathscr{H}} : \pi_1(\mathscr{H}) \to \mathrm{Mod}(\Sigma_g)[\phi]$$

is surjective.

- Have found a large quotient of $\pi_1(\mathscr{H})$
- Leads to a classification of components components of "marked strata" (refining Kontsevich-Zorich)
- Tells us about cylinders

*: this is the generic case

A word on the proof



How to show these twists generate $\mathrm{Mod}(\Sigma_g)[\phi]$?

Nonseparating curve a *admissible* if $\phi(a) = 0$

Admissible subgroup $\mathcal{T}_\phi = \langle T_a \mid a \text{ admissible} \rangle$

Step 1: show your twist-set generates $\mathcal{T}_\phi$

Complex of admissible curves, subsurface point-push subgroups

Step 2: show $\mathcal{T}_\phi = \mathrm{Mod}(\Sigma_g)[\phi]$

Ode to Dennis Johnson. Compare your groups rel.
Johnson filtration. "Artin group relations"

Thank you for your attention. Here is a cat.