MATH 80440: Topics in the braid group
Spring 2024

Meeting Time:	MW 3:30–4:45 PM
Location:	Hayes Healy 125
Instructor:	Nick Salter Office: Hurley 277 Email: `nsalter@nd.edu`
Office hours:	Location: Hurley 277 Tuesday 10 - 12 am
Course Website:	`https://nsalter.science.nd.edu/teaching/braidsspring2024/`

Textbook: For the first couple of weeks we will follow Chapter 9 of *A Primer on Mapping Class Groups* by Farb-Margalit. We will then draw mostly on the primary literature; a detailed list of sources will be maintained on the course webpage.

Plan for the course: We will begin with the basic theory of the braid group as both the fundamental group of a configuration space and as a mapping class group, and establish basic structural properties following the treatment in Farb-Margalit. The remainder of the course will be a sequence of largely-independent modules. Background will be presented as necessary. Some possibilities are listed below, but I also welcome suggestions based on the interest of the students.

- **Braid groups and knot theory:** I will present three foundational results (the theorems of Alexander, Markov, and Birman-Menasco) that together assert that every link in the three-sphere admits a (non-unique) expression as the closure of a braid, and describe a simple set of moves that suffice to pass between any two braid representations of the same link. I will also discuss a couple of results about "positivity": Stallings' theorem that positive knots are fibered, and Rudolph's theory of quasipositive braids.

- **Representations of braid groups:** I will discuss three of the most important representations of the braid group: the representations of Burau, Jones, and Lawrence–Krammer. We will see what each of these tells us about braids and the associated links obtained as their closures.

- **Topology of configuration spaces:** A closer look at the topology of configuration spaces. Cohomology computations, after Arnol'd and Fuchs. Applications to the inherent complexity of root-finding algorithms, a la Smale.