(1) (a) Let X be a compact Riemann surface of genus g, and let $\phi : X \to \mathbb{CP}^1$ be a map of degree 3 with only simple branch points (i.e. where at most two points collide in each fiber). Determine the number of branch points.

(b) Do a naïve dimension count for the space of trigonal curves of genus g for all $g \geq 3$. For which genera does one expect all curves to be trigonal? [Hints: Assignment 7, problem 2 is relevant. Also, be careful in genus $g = 3$!]

(2) Let X be the curve defined by $y^2 = x(x^4 - 1)$.

(a) Exhibit an automorphism $T : X \to X$ of order 8.

(b) Take as given the fact that X can be constructed as a regular octagon with opposite edges identified by translation, with T acting as rotation by $2\pi/8$. Show that there exists $C \in H_1(X; \mathbb{Z})$ such that $H_1(X; \mathbb{Z}) \cong \mathbb{Z}[T]/(T^4 + 1)$.

(c) Exhibit a basis for $\Omega(X)$ consisting of T-eigenvectors.

(d) Show there is an isomorphism $\mathrm{Jac}(X) \cong \mathbb{C}^2/\mathbb{Z}[(\zeta, \zeta^3)]$, where $\zeta = e^{2\pi i/8}$.