As usual, $X$ denotes a compact Riemann surface of genus $g$.

(1) Let $D = P_1 + \cdots + P_d$ be any effective divisor of degree $d < g$. Prove that one can choose $P_i \in X$ for $d < i \leqslant 2g-2$ such that $K = \sum_{i=1}^{2g-2} P_i$ is a canonical divisor.

(2) Let $D = \sum_{i=1}^{d} P_i$ be a fiber of a nonconstant map $f : X \to \mathbb{CP}^1$ of the least possible degree $d > 0$.
   (a) Show that $\dim |D| = 1$. [Hint: If the dimension were any larger, construct a map $g : X \to \mathbb{CP}^1$ of smaller degree by a well-chosen projection.]
   (b) Let $E = K - D + P_1$. Show that $\dim |E| = g - d$, and that the base locus of $E$ contains $P_1$.

(3) Applying the preceding to a suitable hyperelliptic curve, give an explicit example of a complete linear system with nontrivial base locus and $\dim |E| > 0$.

(4) Give some thought this week (and for the rest of the semester!) to picking an advisor/research area. Mathematical things to think about beyond the high-level research area: what sorts of technical arguments do you enjoy, and which do you hate? (e.g. - linear algebra, calculus, inequalities, combinatorics, group theory, etc.) Find out if the sorts of technicalities you'd need to master are a good fit with what you like doing. Also, think about what your advisor is like as a person: do you feel like you can learn effectively from them? Do you feel comfortable around them?