In problems 1-3, X is the hyperelliptic curve defined by $y^2 = p(x)$ for $p(x)$ a polynomial of degree $2g + 2$ with distinct roots.

(1) Exhibit a basis for $\Omega(X)$ (global holomorphic differentials). As usual, each differential should be of the form $\omega = q(x, y)dx$ and/or $\omega = r(x, y)dy$, with $q(x, y), r(x, y)$ rational functions of x, y.

(2) Let $\iota : X \to X$ be the hyperelliptic involution $\iota(x, y) = (x, -y)$. Determine the eigenspaces of ι^* acting on $\Omega(X)$.

(3) Let $P_0 \in X$ be a Weierstrass point, so that $P_0 = (x_0, 0)$ for some root x_0 of p, let $P' = (x, y)$ be any ordinary point (so that $y \neq 0$), and let P_∞ be one of the two points over ∞.
   (a) Compute $h^0(nP_0)$ for all $n \geq 0$.
   (b) Compute $h^0(nP')$ for all $n \geq 0$.
   (c) Compute $h^0(nP_\infty)$ for all $n \geq 0$.

(4) Now let X be an arbitrary compact Riemann surface, and let $D = \sum_{i=1}^{d} P_i$ be a fiber of a nonconstant map $f : X \to \mathbb{CP}^1$ of the least possible degree $d > 0$.
   (a) Show that $\dim |D| = 1$. [Hint: If the dimension were any larger, construct a map $g : X \to \mathbb{CP}^1$ of smaller degree by a well-chosen projection.]
   (b) Let $E = K - D + P_1$. Show that $\dim |E| = g - d$, and that the base locus of E contains P_1.

(5) Applying the preceding to a suitable hyperelliptic curve, give an explicit example of a complete linear system with nontrivial base locus and $\dim |E| > 0$.

(6) Give some thought this week (and for the rest of the semester!) to picking an advisor/research area. Mathematical things to think about beyond the high-level research area: what sorts of technical arguments do you enjoy, and which do you hate? (e.g. - linear algebra, calculus, inequalities, combinatorics, group theory, etc.) Find out if the sorts of technicalities you'd need to master are a good fit with what you like doing. Also, think about what your advisor is like as a person: do you feel like you can learn effectively from them? Do you feel comfortable around them?