(1) Let
$$x^3y + y^3z + z^3x = 0$$
define the *Klein quartic* X, a smooth Riemann surface of genus 3 in $\mathbb{CP}^2$.
   (a) Find nonzero complex numbers α, β, γ so that X admits an automorphism of the form $f([x : y : z]) = [\alpha x : \beta y : \gamma z]$ of order 7.
   (b) Show that each fixed point of f is a Weierstrass point.
   (c) Choose one such point P, and find an explicit meromorphic function on X with a pole just at P.

(2) Let D be an effective divisor on a compact Riemann surface X. Prove that $\dim |D| \geq t$ if and only if for every t points $P_1, \ldots, P_t$ on X, there is $D' \in |D|$ passing through $P_1, \ldots, P_t$.

(3) Let D, E be effective divisors. Prove that
$$\dim |D| + \dim |E| \leqslant \dim |D + E| .$$
[Hint: Use the previous problem. Riemann-Roch will get you nowhere.]

(4) (Clifford's theorem) Let D be a special divisor of degree d. Prove that $\dim |D| \leqslant \frac{d}{2}$. [Hint: apply the preceding to the pair of divisors $D, K - D$ and then invoke Riemann-Roch. Be sure to explain where the hypothesis that D is special is being used.]