[Full disclosure: if you get stuck, consult p. 250 of Griffiths-Harris.]

(1) Let D be an effective divisor on a compact Riemann surface X. Prove that $\dim |D| \geq t$ if and only if for every t points $P_1, \ldots, P_t$ on X, there is $D' \in |D|$ passing through $P_1, \ldots, P_t$.

(2) Let D, E be effective divisors. Prove that

$$\dim |D| + \dim |E| \leqslant \dim |D + E| .$$

[Hint: Use the previous problem. Riemann-Roch will get you nowhere.]

(3) (Clifford's theorem) Let D be a special divisor of degree d. Prove that $\dim |D| \leqslant \frac{d}{2}$. [Hint: apply the preceding to the pair of divisors $D, K - D$ and then invoke Riemann-Roch. Be sure to explain where the hypothesis that D is special is being used.]