Instructions: You may consult any references you like as long as you document what you use. However, do not work with anyone else. Solutions will be graded both for correctness and for clarity of exposition.

Throughout, X denotes the solution set in $\mathbb{CP}^2$ to the equation

$$F(x, y, z) = x^4 + y^4 + z^4 + x^2 z^2.$$

(1) Verify that X is a Riemann surface.

(2) By considering a convenient projection $\pi : X \subset \mathbb{CP}^2 \to \mathbb{CP}^1$, use the Riemann-Hurwitz formula to show that X has genus 3.

(3) Construct a basis of explicit holomorphic differentials for $\Omega(X)$.

(4) In class I asserted that our proof of Riemann-Roch used no more analytic machinery than the Dolbeault lemma and partitions of unity. How true is this? By carefully reviewing the proof of Riemann-Roch given in Section 8 of McMullen's notes, pinpoint the place where Serre Duality (Theorem 7.10) is used in our argument. For bonus credit, explain how to prove Riemann-Roch without invoking Serre duality at this point.